Review Article

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Influence of material uncertainties on the RLC parameters of wound inductors modeled using the finite element method

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Abstract: In this work, we highlight the influence of the material uncertainties (magnetic permeability, electric conductivity of a Mn-Zn ferrite core, and electric permittivity of wire insulation) on the RLC parameters of a wound inductor extracted from the finite element method. To that end, the finite element method is embedded in a Monte Carlo simulation. We show that considering mentioned different material properties as real random variables, leads to significant variations in the distributions of the RLC parameters.

Keywords: ferrite core; finite element; material uncertainties; Monte Carlo simulation; wound magnetic components

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1 Introduction

In most cases in finite element (FE) modeling, some electromagnetic properties of materials are not sufficiently known due to the lack of certain information (*i.e.* uncertainties on data presented in the specification sheets). This is the case of wound magnetic components. Such components are typically used in switch-mode power supplies where the high switching frequencies yields small values of the volume and weight. On the other hand, the fast switching operation also implies the occurrence of undesirable phenomena (skin and proximity effect, influence of the parasitic capacitances, higher magnetic losses, etc.), still challenging to model. Indeed, one of the main limitations in frequency increase is the energy dissipation by losses in ferrite that produces heating of the electronic circuits. Due to their complex structure and related complex electrical behavior, there are few data on the conductivity properties of the sintered Mn-Zn ferrite causing difficulties in correctly modeling the eddy current phenomenon [1]. Also the errors arising in the characterization of soft magnetic materials from measurements, are directly reflected in the mathematical models of material and so resulting in inaccuracies in numerical simulations. Despite recent efforts oriented in this direction [2, 3], some differences are still observed between numerical simulations (through extracted RLC equivalent circuits) and experimental measurements conducted on high-frequency wound magnetic components (see *e.g.* [4]). To some extent, such discrepancies are caused by geometric and material uncertainties. For instance, the magnetic permeability and the electric conductivity of the ferrite magnetic core are usually not known with accuracy less than 20% and 5%, respectively. In addition, the wire diameter may also vary due to the crushing of the surrounding insulating material, and uncertainties in the winding configuration due to the wounding procedure may also arise [5].

The influence of the variability of input parameters of the numerical model on its outputs must be considered. The uses of perturbation method, Monte Carlo simulation, polynomial chaos, etc. are usually performed for this purpose [6, 7]. The perturbation method is based on a Taylor series development of random quantities in the numerical model around their mean value. The calculation of derivatives can be quite complicated and costly, which limits the use of this method only to order 1 and for models having certain regularity. According to the literature, the theory of polynomial chaos is advantageous in the case where several random entries have to be taken into account. On the other hand the Monte Carlo method is simpler to implement but expensive in computing time because it needs several thousand iterations to tend towards a better estimate.

In the present paper, we study the influence of material uncertainties on the RLC parameters of wound inductors using the Finite element method. Since there are

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only three main random inputs (magnetic permeability and electrical conductivity of the ferrite core, and electrical permittivity of the insulation) to be considered in this analysis, we will use the Monte Carlo method to analyze their influences on the RLC parameters of the wound inductor. We present the applied methodology including the stochastic approach and the FE numerical model used to extract the RLC parameters in Section 2. In Section 3, we will describe the different used models for the random inputs including uniform uncertainties distributions along the frequency. Thereafter in the Section 4, the simulation results on a test case will be presented and discussed. Finally in the conclusion, the strengths and the weaknesses of the proposed methodology will be discussed and some perspective works will be proposed.

2 Proposed methodology

In order to obtain the distribution the RLC parameters of the wound inductor due to the propagation of material uncertainties through the numerical model, the Monte Carlo simulation is well known as simple and easy to implement [5–7]. In this stochastic method, the considered input parameters are the magnetic permeability and the electrical conductivity of the ferrite core and the electrical permittivity of the wire insulating material. Both permeability and permittivity are modeled as complex functions of the frequency with the same accuracy along the frequency while the electrical conductivity is taken as real number function of the frequency too. They are all supposed to be uniformly distributed between two limit values computed from the manufacturer's data [8, 9]. These random inputs are propagated through the deterministic numerical model to the outputs parameters (RLC). The implemented FE 2D model of wound inductor is based on two uncoupled problems for computing the RLC parameters. A magneto-dynamic $\boldsymbol{a}$-v formulation with circuit coupling is employed to extract the RL parameters of the inductor [10]. The generated electric potentials of the conductors are then used in an electrostatics problem unlike in many other works (in which a linear repartition of the electric potential along the conductors is a priori assumed). In that way, a weak coupling between the electric and magnetic fields is considered. Besides, massive conductors are considered in order to take into account the skin and proximity effects in conductors. The stray capacitance of the inductor is obtained from the electrostatic energy computed by solving the electrostatics problem in the non conductive materials.

3 Modeling material uncertainties

3.1 Magnetic permeability of the ferrite core

In this study, the ferrite core is considered as magnetic dielectric characterized by its permeability and permittivity. The global approach for modelling different losses in ferrite, consists in representing the electrical permittivity and the magnetic permeability as complex quantities function of the frequency.

$$\mu(\omega) = \mu^{'}(\omega) - i\mu^{''}(\omega) \tag{1}$$

$$\xi(\omega) = \xi^{'}(\omega) - i\xi^{''}(\omega) \tag{2}$$

whith ω, the angular pulsation; $\mu^{'}$, $\xi^{'}$ and $\mu^{''}$, $\xi^{''}$ the real and the imaginary parts of the magnetic permeability and the electrical permittivity respectively.

The real parts represent the magnetic and electric energy storage terms while the imaginary parts, highlight the magnetic and the electric losses respectively. The magnetic losses include the hysteresis and residual losses while the electric losses include Joule losses (due to eddy currents, in the ferrite core for instance) and the dielectric losses associated to the rotation of dielectric dipoles due to the alternative electric field [11].

For the frequency dependence of the complex forms, the Debye relaxation model [12] is sufficient for modelling both permeability and permittivity, of the ferrite core (see (3) and (4)).

$$\mu(f) = \mu_s/(1 + i(f/f_{r\mu})) \tag{3}$$

$$\xi(f) = \xi_{inf} + (\xi_s - \xi_{inf})/(1 + i(f/f_{r\xi})) \tag{4}$$

with μ_s, ξ_s, the static (*i.e.* at zero frequency) permeability and permittivity respectively; ξ_{inf}, the permittivity at infinite frequency; $f_{r\mu}$ and $f_{r\xi}$, the relaxation frequencies of the material properties. For these analytic models, static parameters are considered as random variables for our deterministic model while the relaxation frequencies are derived from data sheet as in other references [13]. The Figure 2 illustrates the evolutions as frequency functions of the real and imaginary parts of the relative magnetic permeability of the $3F35$ ferrite core, with a relaxation frequency located around $5.5e + 6$ Hz. One can deduce that in a limited frequency range (up to 100 kHz), the magnetic permeability could be considered as a constant real parameter.

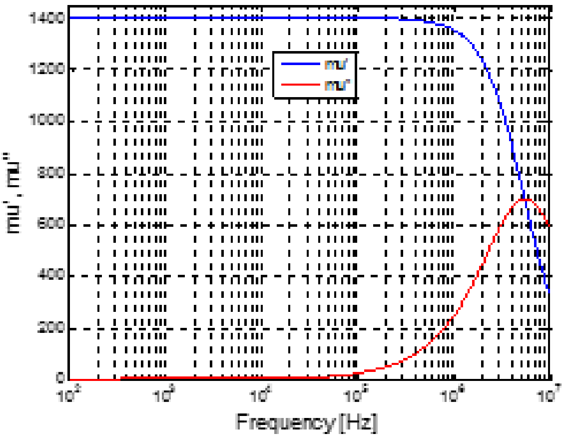


Figure 1: Complex permeability of a Mn-Zn ferrite core as a function of frequency

3.2 Electric permittivity of wire insulating

For the electrical permittivity of the wire insulating material (*i.e.* polyurethane), the Havriliak-Negami relaxation model (empirical modification of Debye relaxation) is the well adapted in this study from the experimental point of view [14, 15]. The real and imaginary parts of the permittivity are taken from the following expression:

$$\xi(\omega) = \xi_{inf} + (\xi_s - \xi_{inf})/[1 + (i\omega\tau_0)^{(1-\alpha)}]^{\beta} \qquad (5)$$

where ξ_s and ξ_{inf} are static (at zero frequency) and infinite frequency permittivity respectively; ω the angular pulsation; $\tau_0 (= 1/(2\pi f_{max}))$ the characteristic time associated to the relaxation frequency f_{max}; and α and β, the shape parameters defining the symmetrical and asymmetrical broadening of the dielectric dispersion curve. From Figure 3, one can see the variation with the frequency of real and imaginary parts of the polyurethane permittivity adopted in this paper. One can notify that decreases monotonically with the frequency and that could be considered as constant on a limited frequency band.

3.3 Electrical conductivity of the ferrite core

The variation of the conductivity with the frequency can be deduced from the imaginary part of the electrical permittivity [12] of the material (either for ferrite or polyurethane) as presented below.

$$\sigma(f) = \sigma_{dc} + \sigma_{ac}(f) \qquad (6)$$

with σ_{dc}, the electric conductivity at zero frequency or dc conductivity; and σ_{ac}, the conductivity at hi frequency.

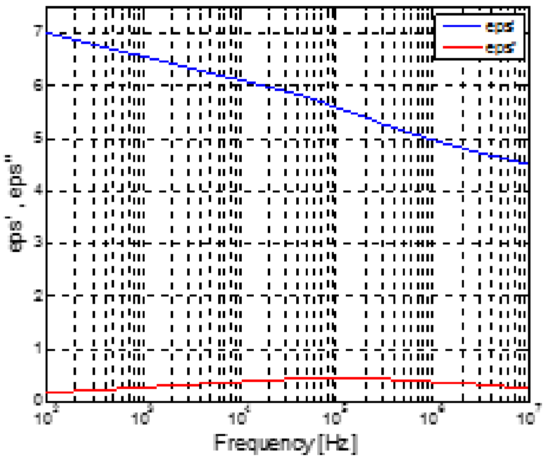


Figure 2: Complex permittivity of polyurethane as a function of frequency

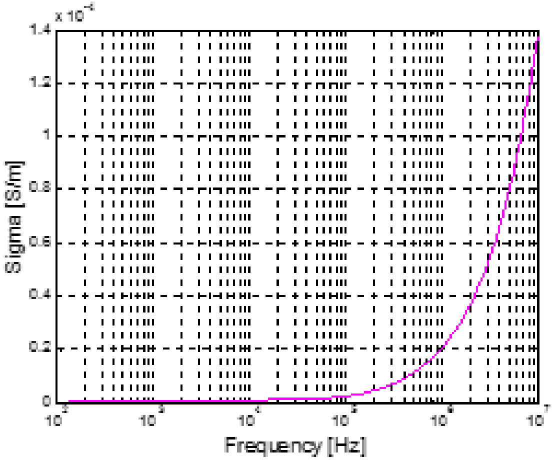


Figure 3: Conductivity versus frequency for ferrite core

This latter is linked to the imaginary part of the permittivity by the following expression:

$$\sigma_{ac} = 2\pi f \xi_0 \xi''(f) \qquad (7)$$

where f is the frequency and $\xi_0 (= 8.854187e - 12\ \mathrm{F/m})$, the vacuum permittivity. The variation with frequency of the conductivity of the polyurethane is depicted on the Figure 4. One can notice that significant Joule losses may appear at hi frequencies, but can be neglected at low frequency.

4 Numerical results

The stochastic study is applied on a 2D axisymmetric model of a wound inductor made of 81-turns and of

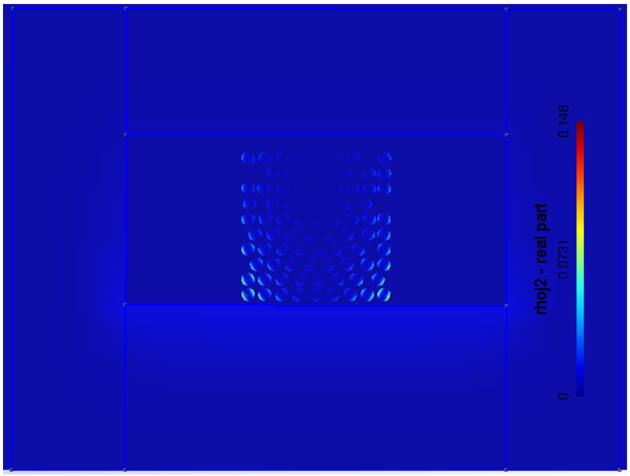


Figure 4: Distribution of Joule losses in the wound inductor at 1 MHz

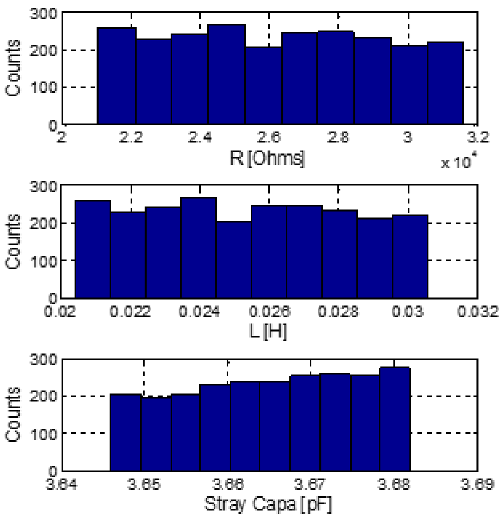
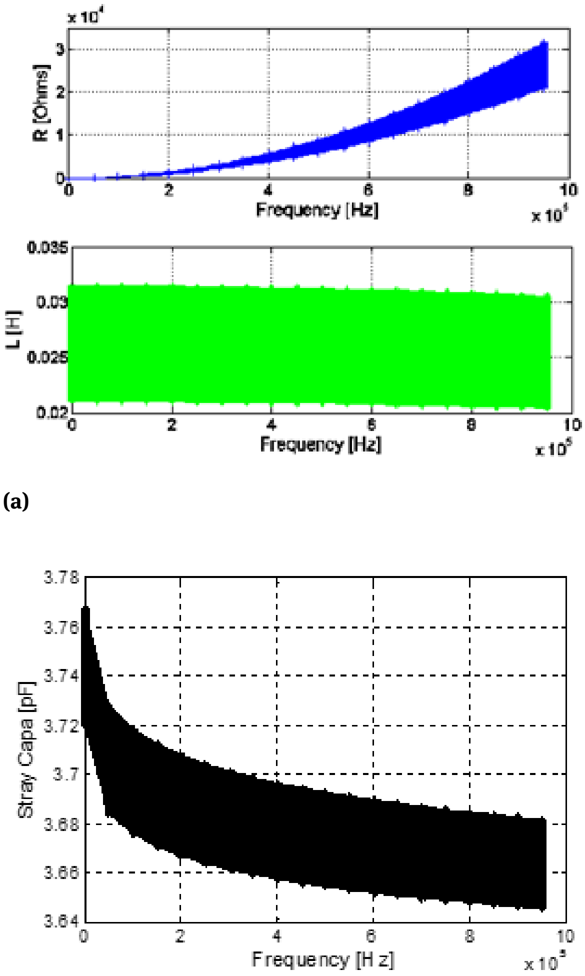


Figure 5: Histograms of RLC parameters at 1 MHz

0.315mm wire diameter. The pattern of conductors in the winding window is chosen so as to reduce the parasitic capacitances of the system. Hence the hexagonal arrangement of conductors in the winding window and the discontinuous type arrangement are then chosen [16]. The evolution of the extracted RLC parameters with frequency, from the Monte Carlo simulation, is depicted in Figure 6. This is the result of Monte Carlo 2350 iterations for which the three random inputs parameters are frequency-dependent and uniformly distributed between two limits computed from data sheets. The increasing dispersion of R with the frequency is due to the skin and proximity effects in conductors and in the ferrite core. The quasi-constant dispersion of inductance along the frequency is due to the fact that the deep down eddy currents (in the ferrite core, see Figure 4), which are shifted in phase and could affect the inductance, are exponentially smaller and so their effects are small or negligible on the distribution of L. While the eddy currents which flow at the exterior surface of the core is in phase with the coil current and then can significantly affect the real part of coil impedance. For cons, the stray capacitances show an increasing dispersion with frequency. This is due to the dispersion of the uncertainties on the insulating material properties (which are taken into account in the capacitance computation) with the frequency.



(a)

(b)

Figure 6: Extracted RLC parameters against frequency

The distributions of RLC parameters at 1 MHz are depicted in the Figure 5. It may be mentioned that they are uniformly distributed around their mean values as the random inputs parameters. For convergence analysis, the adapted mathematical expectations of RLC parameters at 1 MHz with their 95% confidence intervals are depicted in Figure 7. It can be observed here that Monte Carlo simulation achieves a good convergence from 1500 iterations

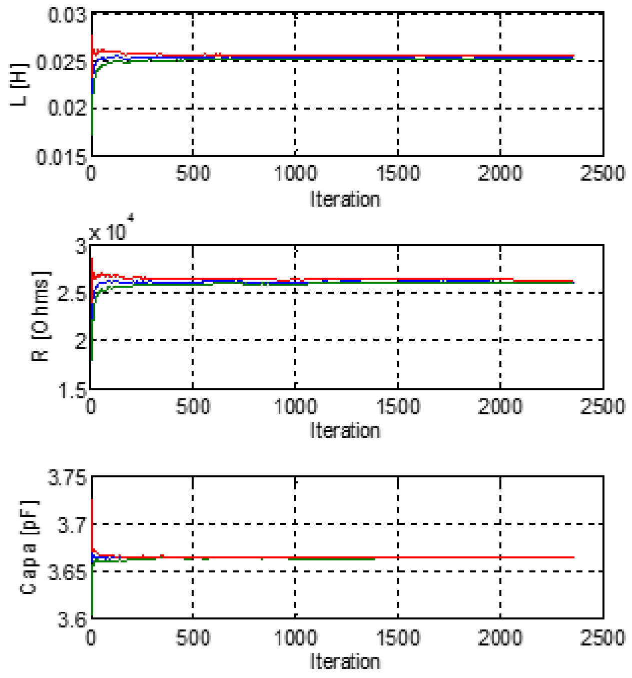


Figure 7: Convergence of RLC parameters at 1 MHz with their confidence intervals

thanks to the used stochastic models of materials uncertainties.

5 Conclusion

In this paper, we have analyzed the influence of material uncertainties (permeability and conductivity of the ferrite core and the permittivity of the wire insulating material) on the extracted RLC parameters of a wound inductor through the FE method. The proposed methodology using MC simulation has been applied successfully to a 2D model including random parameters (material uncertainties only). However, in case of the consideration of geometrical uncertainties (winding pattern and air gap core for example) in addition to the material uncertainties (permeability and conductivity of ferrite core), a mixed stochastic approach implementing for instance chaos polynomials expansions and Monte Carlo simulation, could be interesting in order to benefit from the advantages of both techniques. Another interesting work would be to take into account the material uncertainties in nonlinear behavior laws (*e.g.* B-H curve of ferromagnetic materials) for a sensitivity analysis on the performances of electrical machines for instance. In future works, we plan to compare the impedances computed from the numerical models and ones resulting from experimental measurements.

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